On the Pontryagin Maximum Principle

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Abstract

This talk is based on the following variational condition, together with a Lagrange multiplier rule previously established by the authors.

Let X and Y be Banach spaces, $\varphi : X \times Y \longrightarrow R \cup \{+\infty\}$ be a lower semicontinuous function and $L : Y \longrightarrow X$ be a bounded linear operator and

$$S := \{(Ly, y) \in X \times Y \ : \ y \in Y\}$$

be its graph. It is said that φ satisfies the **variational condition** at $(\bar{x}, \bar{y}, \varphi(\bar{x}, \bar{y}))$ with correcting set $U \subset X \times Y \times R$ iff there exists a positive real $\bar{\delta} > 0$ such that for every $\lambda \in [0, \bar{\delta}]$ the following inclusion holds true:

$$\operatorname{epi} \varphi \cap \left((\bar{x}, \bar{y}, \varphi(\bar{x}, \bar{y})) + \bar{\delta} \cdot \bar{B}_{X \times Y \times R}\right) + \lambda \left(\bar{B}_X, 0, 0\right) \subset \operatorname{epi} \varphi + \lambda U.$$

Consider the optimization problem

$$(OP) \qquad\qquad \varphi(x, y) \to \min \ \ \text{subject to} \ \ (x, y) \in \widetilde{S}$$

Theorem. Under the above assumptions, let $(\bar{x}, \bar{y})$ be a solution of the problem (OP), the functional φ have finite value at $(\bar{x}, \bar{y})$ and satisfy the variational condition with a correcting set U (having the appearance $U = (U_X, U_Y, U_R)$). Let U be bounded and $\mu\left(U_X - L(U_Y)\right) < 1$ where μ denotes the ball measure of noncompactness (in X).

Then there exists a triple $(\xi, \eta, \zeta) \in X^* \times Y^* \times \mathbb{R}$ such that

(i) $(\xi, \eta, \zeta) \neq (\mathbf{0}, \mathbf{0}, 0)$;

(ii) $\zeta \in \{0, 1\}$;

(iii) $\langle \xi, u \rangle \ + \langle \eta, v \rangle \ = 0$ for every $(u, v) \in S$;

(iv) $\langle \xi, u \rangle \ + \langle \eta, v \rangle \ + \zeta w \geq 0$ for every $(u, v, w) \in \widehat{T}_{epi\ \varphi}(\bar{x}, \bar{y}, \varphi((\bar{x}, \bar{y}))$.

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In this talk, applications of this result will be presented. They concern necessary optimality conditions for optimal control problems whose dynamics are governed by differential inclusions in the presence of pure state constraints of inequality type.